METHODOLOGY FOR URANIUM RESOURCE ESTIMATES AND RELIABILITY

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INTRODUCTION

The Grand Junction Office began making estimates of undiscovered uranium resources as early as 1948. The early estimates were necessary to establish a basis for negotiating lease conditions of Government-controlled lands to prospective mining companies. The estimating procedure was based on the principle of geologic analogy: if one area has known ore deposits of certain sizes and grades, other areas with very similar geologic settings may have ore deposits of similar sizes and grades.

Geologic analogy[1] is still the root of modern assessment methodology used by the Grand Junction Office of the U.S. Department of Energy (DOE), but the scope of the assessments has grown to a national survey, for the purposes of providing support for decisions concerning the development of the breeder reactor and the role of nuclear power in the national energy economy. As the use of uranium resource estimates has expanded to influence decisions and decision-makers far removed from the information experts who make the assessments, so has the knowledge "gap" between the decision-makers and the assessors tended to increase. This paper will examine how the 1980 National Uranium Resource Evaluation (NURE) assessment methodology, together with its presentation of results, has been focused to reduce this "gap" by moving more information about the assessments and the assessment process up to the decision makers. In addition, improvements over past methods that have been incorporated into the NURE methodology to enhance the rationality and reliability of the assessment process will be reviewed.

This paper will focus on the considerations that went into the design of the assessment methodology. An overview of the method is shown in Figure 1. For a detailed description of the method, the reader is referred to GJO-111(80), "An Assessment Report on Uranium in the United States of America". The mathematical aspects are discussed in GJBX-165(80), a report by Union Carbide - Oak Ridge mathematicians C. E. Ford and R. A. McLaren.

PRINCIPLES OF THE 1980 NURE ASSESSMENT METHOD

Resource assessments are "area" related phenomena and involve three key judgments beyond the establishment that a particular geographically or geologically bounded area is favorable for uranium deposits. The first is the question of whether or not the area does in fact contain undiscovered uranium deposits; the second, assuming deposits do exist, is the question of how much uranium they contain; the third is the question of what portion of the contained uranium could be recovered under various economic conditions.

It is obvious that none of the above questions can be answered with certainty, yet past assessment methodologies, through a "point-estimate" approach, forced the user/decision-maker to construct a probability distribution about the

FIGURE 1

NURE ASSESSMENT METHODOLOGY

1. Quadrangle data gathering and evaluation
2. Identification of favorable areas
3. Control (Analog) area selection and favorable area classification (probable, possible, or speculative)
4. Subjective assessment of endowment factors and physical characteristics affecting economics
5. Computer calculation of endowment and economic potential for $30/lb, $50/lb, and $100/lb U_3O_8 cost categories
6. Assessor and peer review
7. Aggregation of individual favorable area assessments to regional totals
8. Executive panel review

October 1980

estimates; or worse, to misconstrue the estimates as being quantities of resources certain to be discovered and mined, given enough time.

Uranium Resource Estimates are Random Variables

The foremost change in the 1980 NURE assessment methodology over previous DOE methodologies has been the introduction of subjective probability and the use of random variables in attempting to answer the three questions posed by the assessment process. That is, the methodology formally recognizes that the exact quantity of uranium to be found and recovered from any particular region is inherently unknowable, so that an assessment of it must be a probabilistic statement of the assessor's perception of that quantity.

The fact that the estimates contain elements of subjectivity should not be interpreted as evidence that the method is unscientific. The scientific method has been described[2] as, "a simple and abiding faith in the rationality of nature, leading to the belief that phenomena (such as uranium ore deposits) have a cause." This concept is the essence of geologic analogy. In addition, application of the scientific method calls for a rather special mental attitude, foremost of which is a high reverence for facts. No other uranium resource appraisal program has attempted to gather the quantity or variety of factual information as the NURE program.

Endowment Estimation

A second major change in the NURE assessment methodology is the estimation of endowment, defined as the quantity of uranium contained in undiscovered deposits at a grade of 0.01 percent U_3O_8 and higher. Past assessments estimated economic potential resources directly by using production and economic reserves (of a given cost category) as the analog. The current method uses production and reserves-related mineral inventory as the analog.

This approach is seen as a major improvement, as it allows a separation of geologic judgments from economic judgments and permits each set of judgments to be independently studied. This separation enhances the application of the geosciences to the assessment problem by reducing the complexities introduced by economic considerations, as well as improving the consistency in bringing economic constraints into the process. Endowment is assessed by mathematically combining the four estimated factors defined in Figure 2. The more important aspects considered in the design of the endowment estimating equation are summarized in Figure 3.

The disaggregated estimating equation is employed to offset what psychologists Amos Tversky and Daniel Kahneman identified[3] as an "anchoring bias" in the assessment of subjective probability distributions. That is, people tend to state overly narrow confidence intervals which reflect more certainty than is justified by their knowledge, once they have established in their mind what the "most likely" or "right" answer ought to be.

If the assessment of uranium endowment, U_e, is made directly, we therefore could expect the confidence interval to be too optimistic (overly narrow). By examining the components individually, and mathematically calculating the confidence intervals about U_e, we hope to capture a truer picture of the uncertainty inherent in the estimate.

FIGURE 2

ENDOWMENT ESTIMATING EQUATION

$$U_e = A \times F \times T \times G,$$

where

U_e = conditional uranium endowment in tons U_3O_8 above a cutoff of 0.01 percent U_3O_8,

A = projected surface area of favorable area in square miles,

F = the fraction of A that is underlain by endowment,

T = tons of endowed rock per square mile within A x F, and

G = average grade of endowment, in decimal fraction form.

October 1980

FIGURE 3

ESTIMATION OF URANIUM ENDOWMENT

- Uranium endowment is not directly estimated; the estimate is disaggregated into four factors; area, fraction of area underlain by mineralization, tons per square mile of mineralization, and the average grade of the mineralization. The result is the estimating equation, $U_e = A \times F \times T \times G$.
- The factors are sequentially estimated, with F, T, G being treated as random variables requiring high, low, and most-likely values.
- The factors A, F, T, G are assumed to be statistically independent in calculating the product U_e which is also a random variable.
- The equation was utilized in capturing the assessments of all favorable areas even though the assessor may have preferred to define the factors differently, e.g., number of deposits x size x grade.

October 1980

Disaggregating the equation into four factors is somewhat arbitrary but not capricious. The only practical way of combining factors in a multiplicative equation, such as $U_e = A \times F \times T \times G$, is to assume the factors are independent. If too many factors are utilized, assuming independence becomes difficult to justify. Conversely, if only two random variables are used, a significant anchoring bias may still be reflected. Figure 4 summarizes the assumed properties of the distributions for estimated random variables.

Sequential estimation of the factors in the order A, F, T, and G is preferred over other sequences to combat "anchoring" and other biases. In practice, this is not often achieved, largely because the factor F, which captures judgment about the number and size of deposits, is clearly the most difficult to estimate. The factors T and G are relatively less subjective, as the analog or "control area" information is far more relevant and useful in estimating these factors.

Employment of a single, simple estimating equation has advantages and disadvantages as well. The factors are flexibly defined, allowing the favorable area "A" to be treated as either linear, square, or cubic miles. Even when the assessor prefers to think in parameters different than F, T, and G, the equation is simple enough to "back-in" these values and capture their assessments, if not their exact thought process. Because each assessment has the factors in common, the analysis and reporting of the endowment, and downstream calculation of potential resources, are greatly simplified. While some "gaming" with the equation occurred, the disaggregation of the estimating equation was useful in reducing the anchoring bias in the resulting distributions for estimated uranium.

Probability of Occurrence

Another feature of the NURE assessment methodology not previously employed in DOE uranium assessment activities is the formal separation of judgments concerning the probability of occurrence of deposits from judgments about the quantity of uranium contained in the deposits.

While the presence of characteristics indicating that an area is favorable for uranium deposition is reassuring, it is not conclusive evidence of the existence of uranium deposits. Uranium deposition occurs when a complex set of favorable conditions including a uranium source, a transportation mechanism, a depositional environment, and other factors occur simultaneously or in sequence, and for a sufficiently long period of time, to result in significant uranium concentration. The geologist can confirm that some, or perhaps all, of the necessary conditions were met at one time, yet can seldom say with certainty that a uranium deposit exists without conducting an exploratory drilling program.

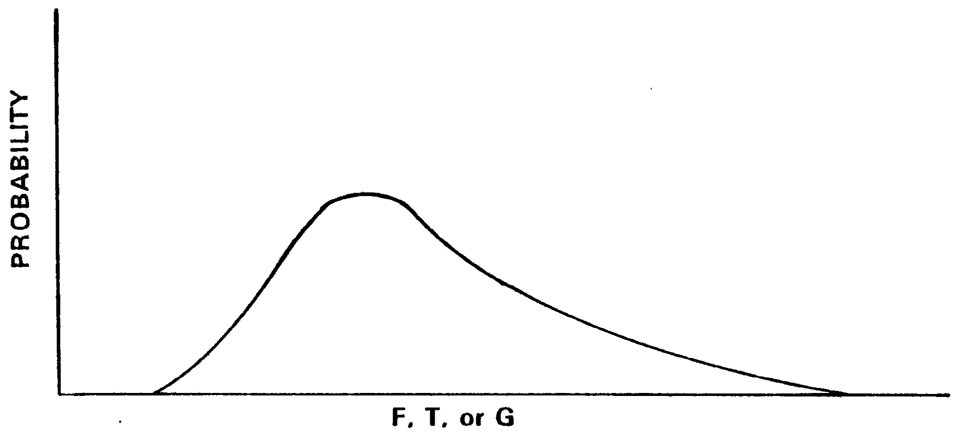
A second justification for separating occurrence from quantity judgments is that it simplifies the estimation of the factors F, T, and G, because it can then be assumed that these are real, positive numbers. It would be very difficult to describe a distribution for the factor F if, for example, the assessor has to subjectively integrate a significant possibility that the true value of F is zero. His "lower" and "most-likely" estimated values might both be zero. He would then be forced to specify several sets of probability and a value of F to describe the tail of the distribution. Tonnage and grade also

FIGURE 4

DISTRIBUTIONS OF THE ESTIMATED RANDOM VARIABLES

Assumptions:

1. Unimodal distribution
2. Postive, negative, or zero skew
3. No probability of zero or less (zero bound)
4. No probability of very large numbers (upper bound)
5. Often similar to log-normal, except:
   a.) positive displacement
   b.) truncated infinite tail



become confused, as these factors have no meaning if the value of F is really zero.

As a consequence, the endowment estimate produced by the estimating equation is considered to be a "conditional" estimate, the condition being the assumption that one or more deposits do exist. A judgment as to the likelihood, or probability, that one or more deposits exist within the boundaries of the favorable area, is also elicited from the estimating geologist. This factor, termed the probability of occurrence, is an important judgment and is utilized in conjunction with the conditional estimate.

The conditional estimate of U_e for a given area might result in a distribution as shown in Figure 5A. Assume for demonstration purposes that the probability of occurrence was rated as 0.8. The unconditional estimate would then be a combination of this judgment and the conditional estimate as shown in Figure 5B. Figure 5C compares the cumulative distribution functions of the two estimates. The mean of the unconditional estimate is the product of the probability of occurrence and the mean of the conditional estimate.

Similarily, all other points on the conditional cumulative distribution curve are lowered with respect to the "probability of exceeding" axis, by the fractional value of probability of occurrence.

The concept of separating judgments of quantity from the "Yes-No" question concerning whether any significant quantity exists is not unique to the NURE methodology. It was also employed by the U.S. Geological Survey in appraisal of undiscovered oil and gas[4].

Computerized-Cost Models

With the separation of economics from geologic judgments came the opportunity to capture the economic considerations about potential resources in a central model and to apply the economics consistently to all geologic assessments. This is a major improvement over past activities, as it has relieved the estimating geologist from collecting and analyzing cost data and attempting to adjust those data to current dollar values or to specific sets of conditions anticipated in the favorable area.

At the same time, the models are recognized to reflect general cost conditions which may not be appropriate for specific favorable areas, such as remote areas of Alaska or unusual topographic or geologic conditions. Therefore, cost figures produced by the models can be revised by the assessor under such conditions.

Computerizing the models also has accommodated increased complexity in the calculation procedures. For example, mill recovery is a function of the grade of ore fed to the mill, yet it is necessary to assume a mill recovery in establishing an initial mining cut-off grade. In the past, mill recovery was subjectively estimated; now it can be computed with an iterative technique that simultaneously considers the interplay between cut-off grade, average grade, and mill recovery.

The mathematical expressions used in the cost models are summarized in Attachment A. Figures 6A and 6B provide a tabulation of the cost equations

FIGURE 5

CONDITIONAL AND UNCONDITIONAL ESTIMATES

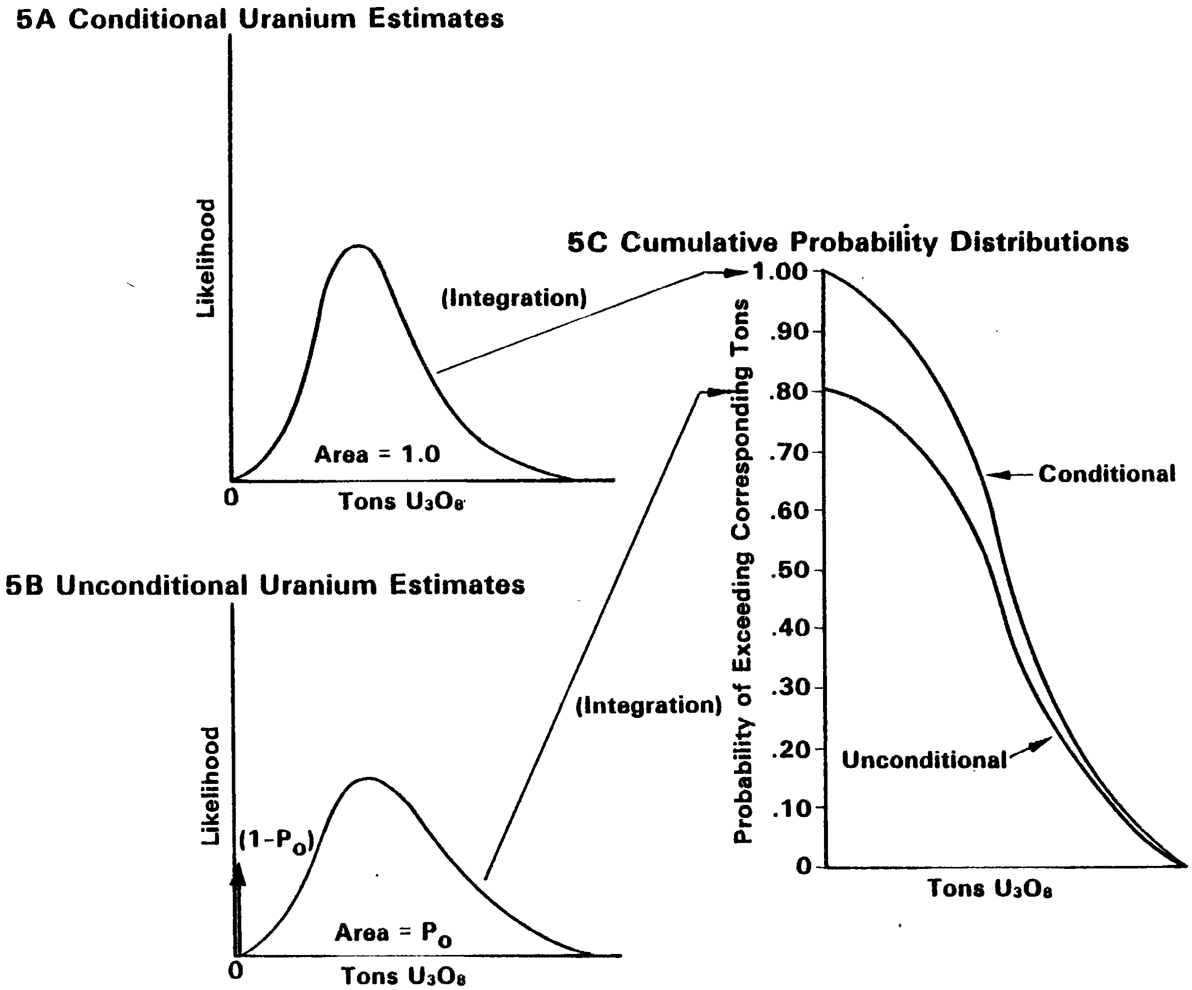


FIGURE 6A

COST FACTOR GENERATOR (Version 101)

A. The Cost Equations:

CAP = AQ + EXPL + DEV DRIL + MINE CAP + MIL CAP (\$/TON)

OP = MINE OP + HAUL + MINE OP + ROYLTY + AVT + SEV (\$/TON)

B. The Variables:

Independent (Geologist Specified) Variables	Model (Internal) Variables	Calculated Cost Variables (\$/ton)	
Resource class	Land costs	CAP	= Total capital costs
Region	Drill spacing	OP	= Total operating costs
State	Wild cat ratios	AQ	= Acquisition costs
Land type	Mill life	EXPL	= Exploration costs
Control area	OP days/yr	DDRL	= Development drilling costs
A, F, T	Cost indices	MINE CAP	= Mine capital
Mine type	Tax, royalty schedules	MILL CAP	= Mill capital
Depth interval	Cost categories	MINE OP	= Mine operating costs
* Mill throughput	Avg grade/cost cat.	HAUL	= Haulage costs
* Mill recovery	U_3O_8 price	MILL OP	= Mill operating costs
		ROYLTY	= Royalty costs
		AVT	= Ad valorem taxes
		SEV	= Severance taxes

*may be defaulted

October 1980

FIGURE 6B

COST FACTOR GENERATOR (VERSION 101)

SUMMARY OF COST FACTOR DETERMINANTS

Independent Variables

Calculated Costs		1 Region	2 Mine Type	3 Depth Interval	4 Mill Size	5 Land Type	6 State	7 A,F,T	8 Resource Class	9 Cost Category
CAP	AQ							X	X	
	EXPL			X				X		
	DDRL			X				X		
	MINE	X	X	X						X
	MILL				X					
OP	MINE	X	X	X						X
	HAUL	X	X							
	MILL	X	X		X					
	ROYLTY		X			X	X			X
	AVT					X	X			X
	SEV					X	X			X
	MINE LOSS	X	X							

October 1980

and the relationships between the geologists' judgments, the internal model variables, and the computed costs.

The models described in the attachment are labeled Version 101, with the implication that they might be revised in the future. The models are quite general and simplistic in their present form. They also are currently deterministic, but work is underway to investigate treating cost factors as random variables.

Calculation of Economic Potential Resources

The calculation of economic potential resources involves an alteration of the endowment equation:

$$U_e = A \times F \times T \times G \text{ (endowment equation)}$$

to

$$U_{Ci} = A \times F \times T \times F(C) \times g \text{ (potential at cost index Ci)}$$

The details involved in substituting F(C), the fraction of rock having grades exceeding the required cut-off grade, and g, the average grade of this fraction, for the average grade of endowment, G, are discussed in Attachment B.

The key assumption underlying the substitution process is that undiscovered deposits will have similar tonnage-grade relationships to those known to exist in the control area deposits from mineral inventory analyses of borehole data.

To clarify, consider Figure 7, which is a histogram of tonnage versus grade increment for known deposits in the Ambrosia Lake, New Mexico, area. The average grade of all material above a cut-off grade of 0.01 percent U_3O_8 for all deposits in the area is 0.07 percent U_3O_8, the same grade as "G" being estimated in the endowment equation.

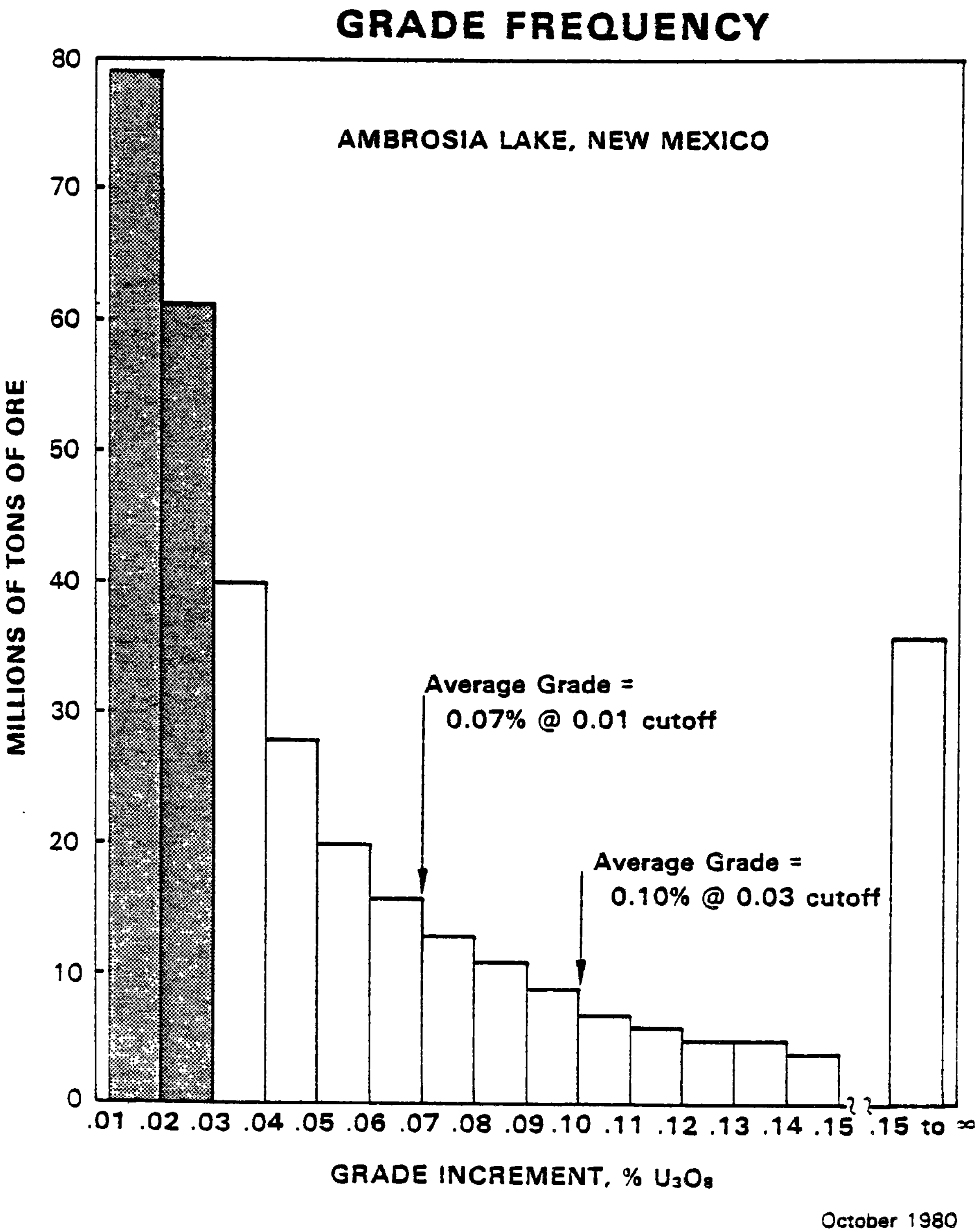
The essence of the economic calculation procedure is to convert judgments about the physical characteristics of the undiscovered orebodies into cost factors, and to combine cost factors to predict both cut-off and minimum average grades according to the following equations:

$$\text{Cut-off grade} = \frac{\text{Operating Costs}}{20 \times \text{Cost Category} \times \text{Mill Recovery}}$$

$$\text{Minimum average grade} = \frac{\text{Operating + Capital Costs}}{20 \times \text{Cost Category} \times \text{Mill Recovery}}$$

As shown in Figure 7, a cut-off grade of 0.03 percent U_3O_8 might be estimated for $50 per pound U308 potential resources, if operating costs are $27/ton of ore. Ore with grades lower than 0.03 percent would be presumed to be uneconomic. The fraction that is economic, F(C), is the area of the histogram to the right of the cut-off grade divided by the area of the entire histogram. The economic material would have a new average grade, g, of 0.10 percent U_3O_8.

FIGURE 7


GRADE FREQUENCY
AMBROSIA LAKE, NEW MEXICO
MILLIONS OF TONS OF ORE
80
70
60
50
40
30
20
10
0
Average Grade =
0.07% @ 0.01 cutoff
Average Grade =
0.10% @ 0.03 cutoff
.01 .02 .03 .04 .05 .06 .07 .08 .09 .10 .11 .12 .13 .14 .15 .15 to ∞
GRADE INCREMENT, % U_3O_8
October 1980

However, in applying the histogram as a control area to undiscovered deposits, we recognize that we cannot predict average grade with certainty, but that it has been described by the assessor with a subjective probability distribution. As a consequence, we do not know the underlying histogram of grade versus tonnage either. If the average grade is really lower than 0.07 percent, the histogram may be smoothed to look like Curve A in Figure 8. If it is higher, it may look like Curve B. The problem is how to construct Curves A and B, given an average grade. This is accomplished by assuming that the cutoff grade versus average grade relationship defined by the control area histogram is valid, even if the yet to be discovered deposits turn out to have a different average grade of all material above 0.01 percent U_3O_8 than the control area.

This assumption, together with the introduction of a computer into the calculation process, makes it possible to acknowledge that the tonnage-grade histograms of undiscovered deposits are not necessarily identical to the one known for the control area. Without a computer, the only practical way to proceed would be to make the very restrictive assumption that all undiscovered deposits would have the same average grade and, therefore, the same normalized tonnage-grade histogram as the control area deposit(s).

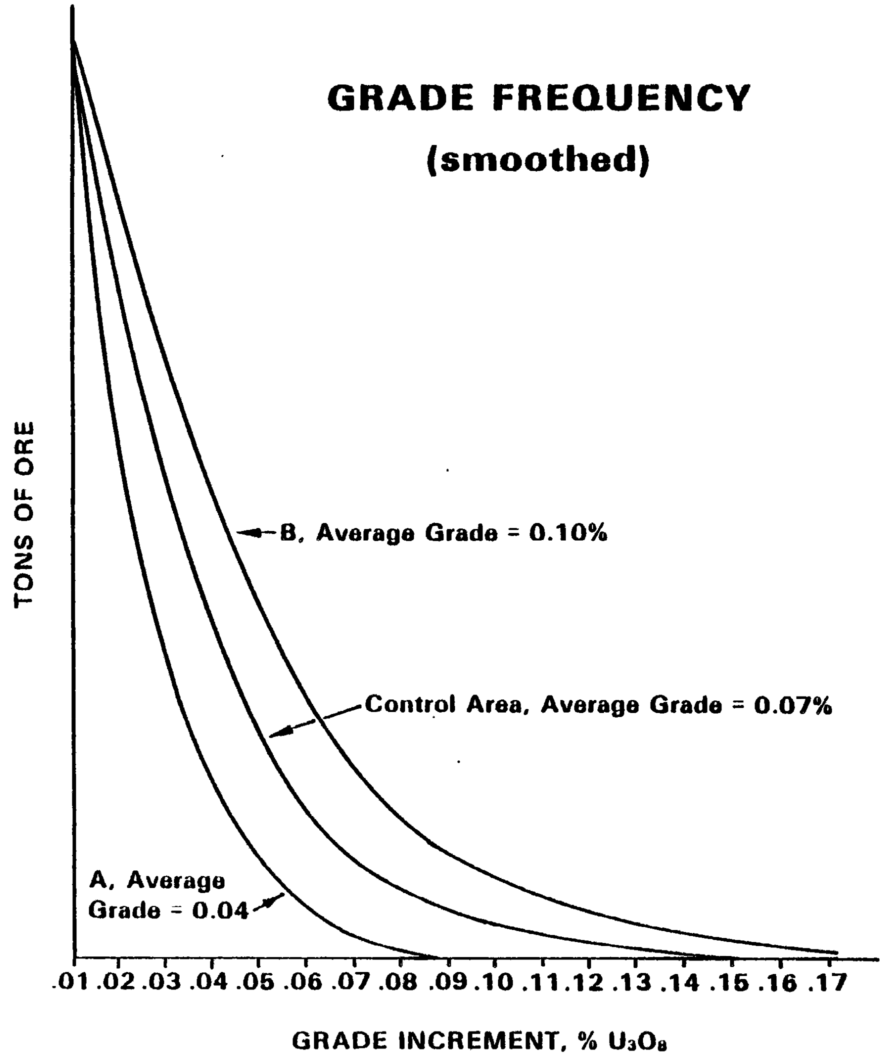
Aggregation of Local Assessments to Regional/National Totals

Since individual estimates of uranium endowment and potential resources for a given cost category are random variables, aggregating them to obtain regional or national totals raises an important issue. The issue concerns the relationships, or dependencies, among the various estimates. If it is assumed that all the estimates are completely unrelated, i.e., statistically independent, the distribution of their sum will reflect the minimum possible uncertainty about the total resources. At the other extreme, if the estimates are assumed to be perfectly correlated with each other, as when each area is identical to every other in all respects, the distribution of the sum will reflect the maximum possible uncertainty, as an error in one estimate would be repeated in all estimates.

The nature of the NURE estimates is such that using either of these simplistic assumptions singly does not appear justified. Even though a relatively large number of geologists were involved in the 1980 NURE assessment activity (about 120 principal investigators) and 73 control areas were defined, the use of a common methodology and estimating equation in developing 646 assessments suggests that some of the estimates are associated with, or partially dependent upon, other estimates for areas having similar geology and other related factors. Alternatively, assuming perfect correlation among all the estimates is very unrealistic, because of the variety of host rocks and expected deposit types involved. Hence, the NURE methodology has adopted a procedure that recognizes that the estimates are "moderately correlated".

The procedure used for aggregating the estimates is to first combine similar estimates (e.g., those made with the same control area) into subgroups as if they are perfectly correlated. This tends to overstate the uncertainty about the sum for each subgroup, because assessments which have "control area" in common may involve different host rocks units, different characteristics of favorability, and even different assessors, and therefore are not "perfectly" correlated. The subgroups are then summed as if they are independent. This

FIGURE 8



October 1980

tends to understate the uncertainty (by ignoring any relationship between groups) but offsets some of the overstated uncertainty generated in the formation of the subgroups. The procedure results in an approximation to a sum of "moderately correlated" estimates which appears to be more reasonable than using either the "independent" or "perfectly correlated" assumptions. As we do not know, nor can determine, the precise relationship of each estimate to every other, it is necessary to employ an approximation technique. Fortunately, the "means" or expected values are additive, whether or not the estimates are independent or correlated, so the best single value estimate is not affected by correlation considerations.

The effect of the "moderately correlated" assumption, as opposed to independent or perfectly correlated, is illustrated in Figure 9. The point where the curves intersect is the expected value (mean) of probable potential resources at $50 per pound U_3O_8, 1.426 million tons U_3O_8.

Various "moderately correlated" assumptions could have been made, as illustrated in Figure 10. The more restrictive the assumptions that form the initial groupings, the closer the resulting "moderately correlated" curve is to the independent sum. It is interesting to note that, no matter how the groups are formed, provided a reasonable number of independent groups are identified, the cumulative distribution of the sum is much closer to the "independent" sum than to the perfectly correlated sum. As demonstrated by Figure 10, the cumulative distributions for national totals are not extremely sensitive to "reasonable" correlation assumptions.

Closing the Information Gap

The use of cumulative probability distribution functions to summarize regional and national assessments, as shown in Figure 11 is a consequence of three closely-related objectives of the NURE methodology:

1. To provide user/decision-makers with more information about the assessments, particularly the aspect of uncertainty

2. To make the uncertainty visible

3. To relieve user/decision-makers of some of the burden of assigning uncertainty to the assessments.

To this end, the 1980 Uranium Assessment report, GJO-111(80), displays about 600 cumulative distribution functions for reserves and resources, by various cost categories and for each region of the Nation. The primary advantages of the cummulative distribution function are shown in Figure 12.

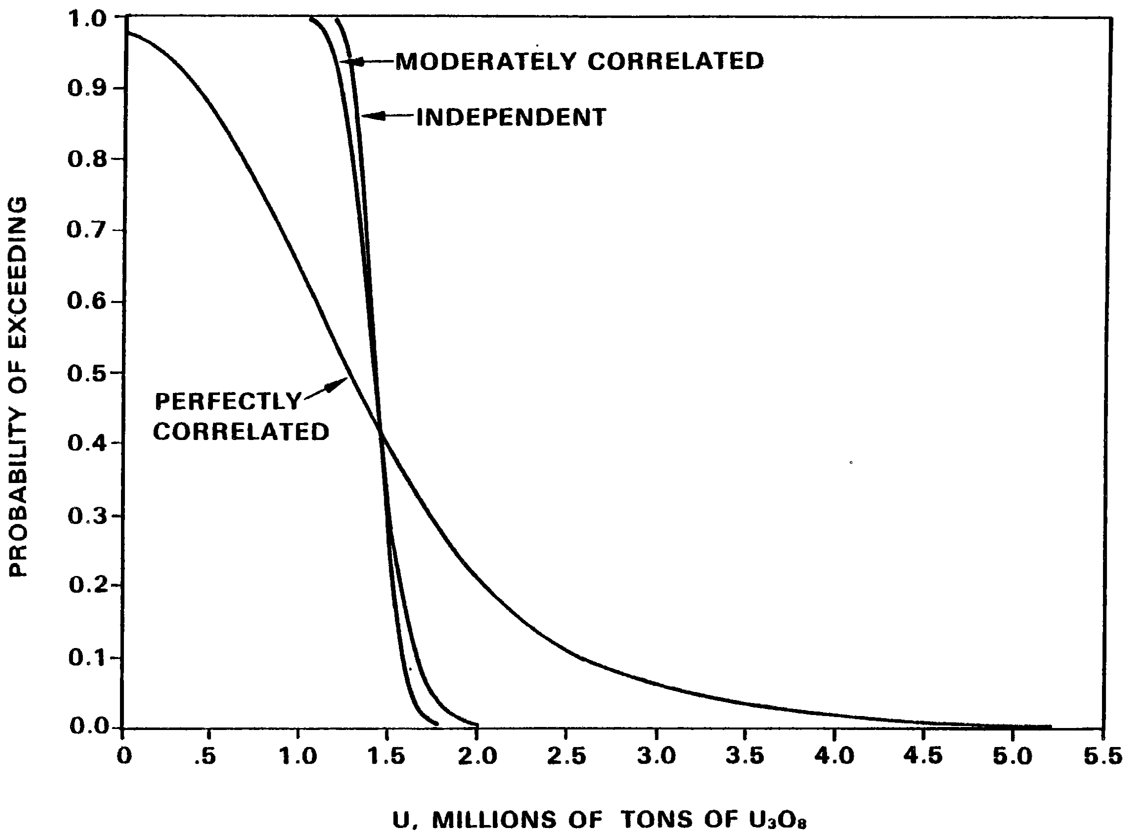
SUMMARY

The NURE uranium assessment method has evolved from a small group of geologists estimating resources on a few lease blocks, to a national survey involving an interdisciplinary system consisting of the following:

a) geology and geologic analogs
b) engineering and cost modelling
c) mathematics and probability theory
d) psychology and elicitation of subjective judgments
e) computerized calculations, computer graphics, and data base management.

FIGURE 9

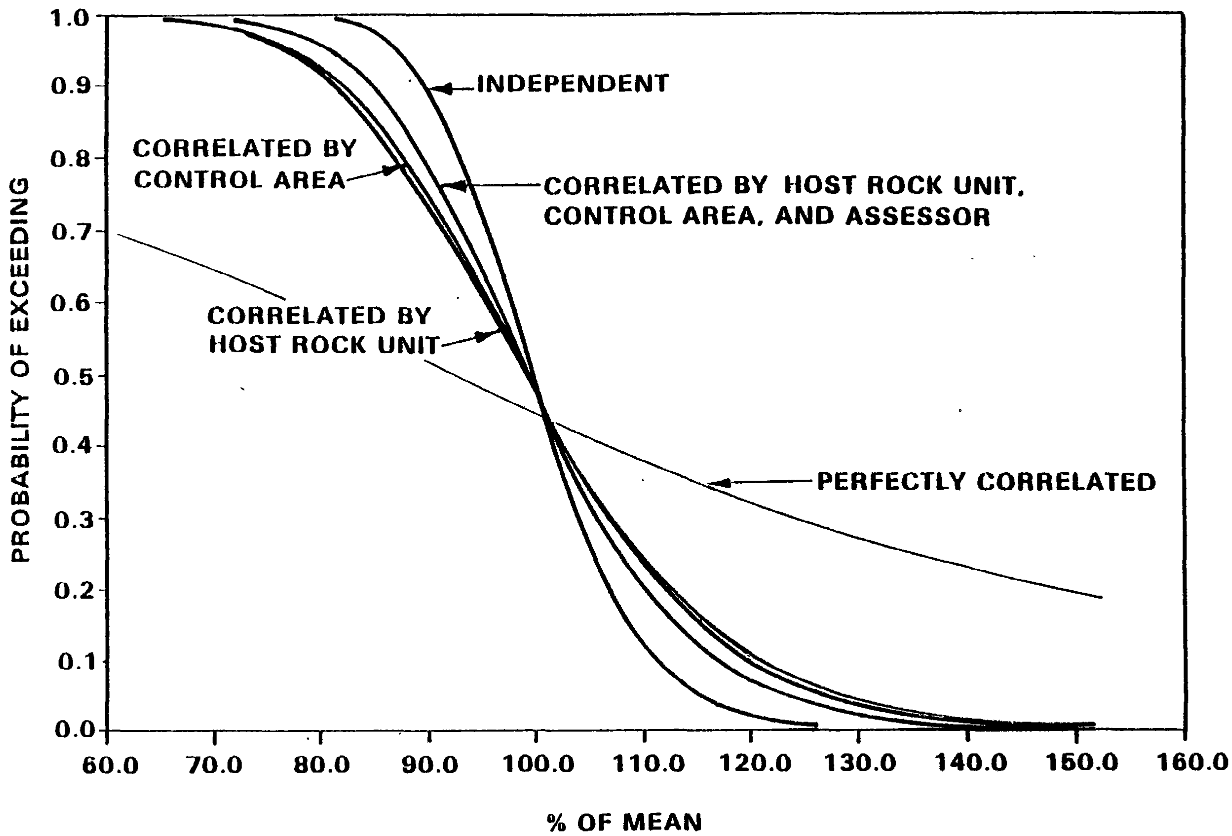
EFFECT OF CORRELATION ASSUMPTION
$50/lb U_3O_8 PROBABLE POTENTIAL
TOTAL U.S.



October 1980

FIGURE 10

EFFECT OF PARTIAL CORRELATION ASSUMPTION $50/lb U_3O_8 PROBABLE POTENTIAL TOTAL U.S.



October 1980

FIGURE 11

RESOURCE CUMULATIVE PROBABILITY DISTRIBUTIONS; RESOURCE CLASSES BY COST CATEGORY

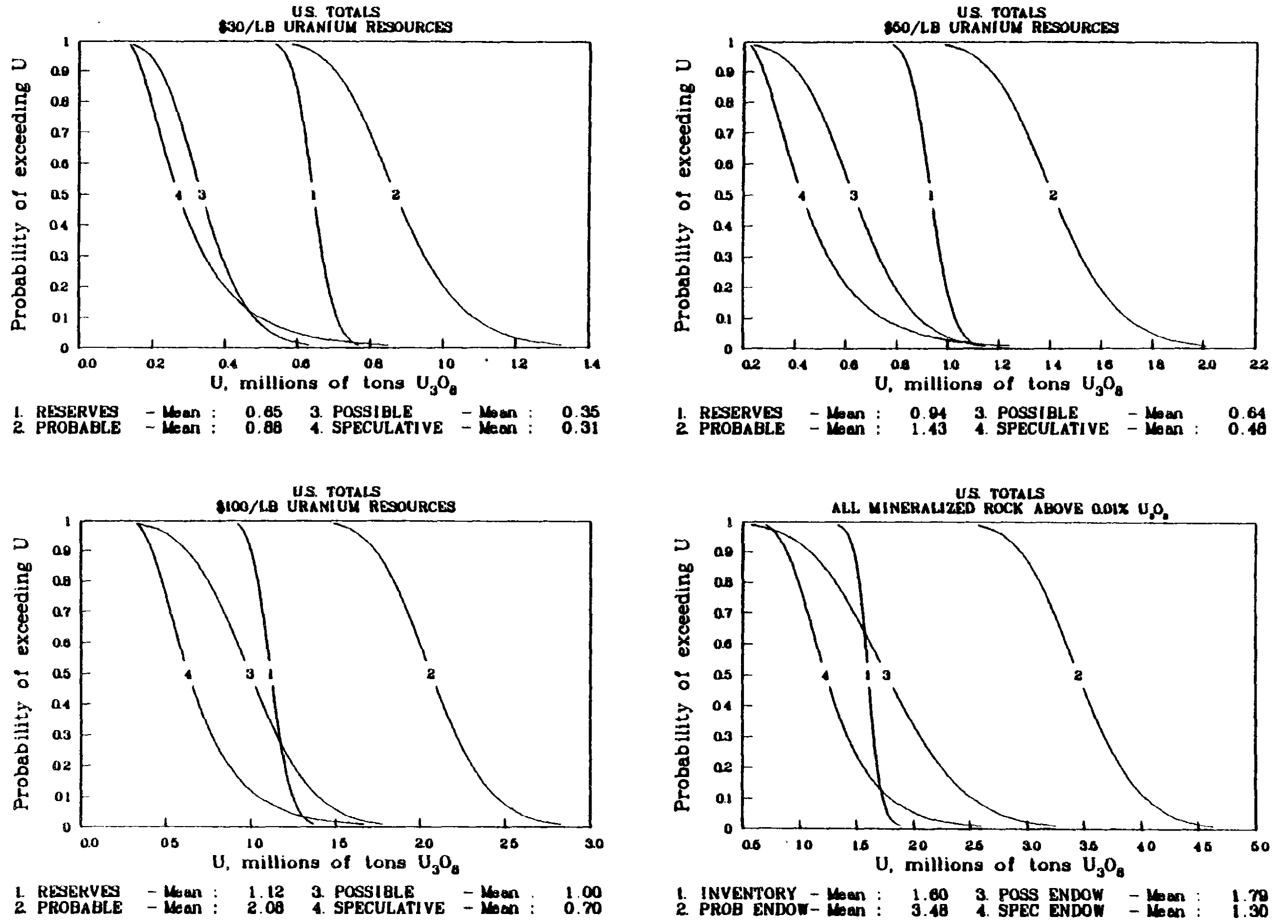


FIGURE 12

ADVANTAGES OF CUMULATIVE DISTRIBUTION FUNCTIONS

- Exposes all "Confidence Intervals", allowing user to select the interval
- Discloses uncertainty, without tables of often confusing statistical factors
- Useful in displaying sums, which need to be computed by GJO to properly recognize dependencies
- Shows reserve estimates and the inclusion of "Higher Risk" resources at a glance
- Reveals relationship of endowment (mineral inventory) to all cost categories
- Amenable to production by computer graphics

October 1980

The evolution has been spurred primarily by two objectives: 1) quantification of uncertainty, and 2) elimination of "simplifying" assumptions. This has resulted in a tremendous data gathering effort and the involvement of hundreds of technical experts, many in uranium geology but many from other fields as well. The rationality of the methods is still largely based on the concept of an analog and the observation that the results are "reasonable".

The reliability, or repeatability, of the assessments is reasonably guaranteed by the series of peer and superior technical reviews which has been formalized under the current methodology. The optimism or pessimism of individual geologists who make the initial assessments is tempered by the review process, resulting in a series of assessments which are a consistent, unbiased reflection of the facts.

Despite the many improvements over past methods, several objectives for future development remain, primarily to reduce subjectivity in utilizing factual information in the estimation of endowment, and to improve the recognition of cost uncertainties in the assessment of economic potential.

The 1980 NURE assessment methodology will undoubtedly be improved, but the reader is reminded that resource estimates are and always will be a forecast for the future. As David B. Hertz once said "When all is said and done, the future is still the future. Therefore, however well we forecast, we are still left with the certain knowledge that we cannot eliminate all uncertainty."[5]

ENDNOTES/REFERENCES

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2. Cyril C. Herrmann and John F. Magee, "Operations Research for Management", Harvard Business Review, July-August 1953.

3. Amos Tversky and Daniel Kahneman, "Judgment Under Uncertainty: Heuristics and Biases", Science, Vol. 185, 27 September 1974, p. 1124-1131.

4. Betty Miller, et.al., Geological Estimates of Undiscovered Recoverable Oil and Gas Resources in the United States, Geological Survey Circular 725, 1975.

5. David B. Hertz, "Risk Analysis in Capital Investment", Harvard Business Review, January-February 1964.

ATTACHMENT A

COST FACTOR GENERATOR (Version 101)

A. <u>The Cost Equations:</u>

CAP = AQ + EXPL + DEV DRILL + MINE CAP + MILL CAP ($/TON)

OP = MINE OP + MILL OP + ROYLTY + AVT + SEV ($/TON)

B. <u>The Variables:</u>

INDEPENDENT (Geologist Specified) Variables	MODEL (Internal) Variables	Calculated Cost Variables ($/TON)
Resource Class	Land Costs	CAP
Region	Drill Spacing	OP
State	Wild Cat Ratios	AQ
Land Type	Mill Life	EXPL
Control Area	OP Days/Yr	DDRL
A,F,T	Cost Indices	MINE CAP
Mine Type	Tax, Royalty Schedules	MILL CAP
Depth Interval	Cost Categories	MINE OP
*Mill Size	Avg Grade/Cost Cat.	HAUL
	U_3O_8 Price	MILL OP
		ROYLTY
		AVT
		SEV

*May be defaulted.

C. <u>Summary of Cost Factor Determinants:</u>

Independent Variables

Calculated Costs		1 Region	2 Mine Type	3 Depth Interval	4 Mill Size	5 Land Type	6 State	7 A,F,T	8 Resource Class	9 Cost Category
CAP	AQ							X	X	
	EXPL			X				X		
	DDRL			X				X		
	MINE	X	X	X						X
	MILL				X					
OP	MINE	X	X	X						X
	HAUL	X	X							
	MILL	X	X		X					
	ROYLTY		X			X	X			X
	AVT					X	X			X
	SEV					X	X			X
	MINE LOSS	X	X							

D. The Cost Models:

I. Acquisition Cost Model

Class 1: $$AQ = \frac{\text{LAND1} \times 640 \times \text{AWCR1}}{\overline{F} \cdot \overline{T}} \times \text{AWCR1}$$.

Class 2: $$AQ = \frac{\text{LAND2} \times 640}{\overline{F} \cdot \overline{T}} \times \text{AWCR2}$$

Class 3: $$AQ = \frac{\text{LAND3} \times 640}{\overline{F} \cdot \overline{T}} \times \text{AWCR3}$$

(In version 101)

LAND1 = $4.80/acre, AWCR1 = 1
LAND2 = $4.80/acre, AWCR2 = 1
LAND3 = $4.80/acre, AWCR3 = 1

1) LAND = average cost of acquired land, $/acre, 1980$
2) AWCR = acquisition wild cat ratio, the average number of times land is acquired before a discovery is made
3) $\overline{F}$ = mean fraction of area underlain by endowment
4) $\overline{T}$ = mean tonnage of endowed rock per square mile

II. Exploration Cost Model

$$\text{EXPL} = \frac{2.015 \cdot \text{WPI79} \cdot F(\overline{F}) \cdot (\text{DEPTH})e^{5.78E-4(\text{DEPTH})} \cdot \text{EWCR}}{\overline{F} \cdot \overline{T}}$$

$\overline{F}$	$F(\overline{F})$
$\geq$E-2	10 holes
E-3	15 holes
E-4	20 holes
E-5	25 holes
$\leq$E-6	30 holes

1) WPI79 = wholesale price index relative to 1/1/79
2) $F(\overline{F})$ = average number of holes per square mile in exploration phase
3) DEPTH = average depth of drilling, feet
4) EWCR = exploration wild cat ratio, 1.0 (in version 101)
5) $\overline{F}$ = mean fraction of area underlain by endowment
6) $\overline{T}$ = mean tonnage of endowed rock per square mile

Note: Both x (lower case) and • will be used as appropriate to indicate multiplication.

E. Mining Losses

The economic calculation is designed to estimate the probability that some portion of the presumed in-place endowment will have a high enough average grade to be recoverable at the stated costs. Historically, all of the ore has not been recoverable, due to physical constraints of mining. It appears appropriate to recognize this fact formally in the calculation of potential resources. Therefore, the following losses for mining operations will be applied in a final calculation procedure.

Mine Type	Mining Loss
Open Pit	3%
Underground	15%
Solution Mine - Wyoming	50%
Solution Mine - Texas	35%

F. Summary of Cost Indices and Other Internal Variables (version 101)

WPI, (Industrial Commodities)

WPI80 = 1.0
WPI79 = 252.8/217.2 = 1.164
WPI78 = 252.8/200.0 = 1.264
WPI77 = 252.8/188.4 = 1.342

MSI, Marshall & Swift Mining-Milling Equipment Cost Index

MSI80 = 1.0
MSI79 = 242/221 = 1.095
MSI78 = 242/203.3 = 1 190
MSI77 = 242/195 = 1.241

CEP, Chemical Engineering Plant Cost Index

CEP80 = 1.0
CEP79 = 250/206 = 1.214
CEP78 = 250/191.7 = 1.304
CEP77 = 250/180 = 1.389

UPRICE = $50/lb
ERDA = $50/lb
GRADE 30 = 0.0015
GRADE 50 = 0.0010
GRADE 100 = 0.0005

III. Development Drilling Cost Model

$$DDRL = \frac{1.788E7 \cdot WPI79 \cdot (DEPTH)e^{5.78E-4(DEPTH)} \cdot DDWCR}{R^2 \cdot \overline{T}}$$

Open pit, solution mine: R = 50 feet
Underground mine: R = 100 feet

1) WPI79 = wholesale price index relative to 1/1/79
2) DEPTH = average depth of drilling, feet
3) DDWCR = development drilling wild cat ratio, 1.0 (in version 101)
4) R = drill spacing, feet
5) $\overline{T}$ = mean tonnage of endowed rock per square mile

IV. Open Pit Mine/Mill Cost Models (1/1/79 Cost Base)

Warm regions: 01,05,06,07,14,15,16; all others cold

Cost		Warm Regions	Cold Regions
MINE CAP	\$30:	$\frac{MSI79}{(6.465E-2 - 7.986E-5 \cdot DEPTH)}$	$\frac{MSI79}{(5.818E-2 - 6.953E-5 \cdot DEPTH)}$
	\$50:	$\frac{MSI79}{(8.409E-2 - 1.012E-4 \cdot DEPTH)}$	$\frac{MSI79}{(7.640E-2 - 9.120E-5 \cdot DEPTH)}$
	\$100:	$\frac{MSI79}{(9.805E-2 - 1.219E-4 \cdot DEPTH)}$	$\frac{MSI79}{(8.850E-2 - 1.081E-4 \cdot DEPTH)}$
MINE OP	\$30:	$\frac{DEPTH \cdot WPI79}{(1.860E-2 \cdot DEPTH + 14.682)}$	$\frac{DEPTH \cdot WPI79}{(1.965E-2 \cdot DEPTH + 12.313)}$
	\$50:	$\frac{DEPTH \cdot WPI79}{(2.729E-2 \cdot DEPTH + 13.935)}$	$\frac{DEPTH \cdot WPI79}{(1.738E-2 \cdot DEPTH + 14.790}$
	\$100:	$\frac{DEPTH \cdot WPI79}{(2.113E-2 \cdot DEPTH + 17.784)}$	$\frac{DEPTH \cdot WPI79}{(2.417E-2 \cdot DEPTH + 14.315)}$

MILL CAP $2.7367 \cdot (MSI79) \cdot (MTHRU)^{-0.2919}$

MILL OP $13.747 \cdot (WPI79) \cdot (MTHRU)^{-0.5778}$

1) DEPTH in ft
2) MTHRU in 1000's tons/day
3) MSI79 = Marshall & Swift Index relative to 1/1/79
4) WPI79 = Wholesale Price Index relative to 1/1/79

V. Underground Mine/Mill Cost Models (1/1/79 Cost Base)

Cost	Cost Category	
MINE CAP	\$30	$\frac{MSI79}{(0.1403 - 2.269E{-5} \cdot DEPTH)}$
	\$50	$\frac{MSI79}{(0.1539 - 2.485E{-5} \cdot DEPTH)}$
	\$100	$\frac{MSI79}{(0.1626 - 2.629E{-5} \cdot DEPTH)}$
MINE OP	\$30	$3.703 \cdot WPI79 \cdot (DEPTH)^{0.351}$
	\$50	$3.890 \cdot WPI79 \cdot (DEPTH)^{0.328}$
	\$100	$3.248 \cdot WPI79 \cdot (DEPTH)^{0.342}$
MILL CAP		$2.7367 \cdot (MSI79) \cdot (MTHRU)^{-0.2919}$
MILL OP		$13.747 \cdot (WPI79) \cdot (MTHRU)^{-0.5778}$

1) DEPTH in ft
2) MTHRU in 1,000's tons/day
3) MSI79 = Marshall & Swift Index relative to 1/1/79
4) WPI79 = Wholesale Price INdex relative to 1/1/79

VI. Solution Mining Cost Models (1/1/80 Cost Base)

Regions 06,07,08,14, and 15 use Texas Model

All other regions use Wyoming Model

	Wyoming Model	Texas Model
MINE CAP	$5.42 = 9.65E-3 \cdot (DEPTH) \cdot (MSI80)$	$2.36 + 7.9533E-3 \cdot (DEPTH) \cdot (MSI8)$
MILL CAP	$\frac{9.854E6 \cdot (MTHRU)^{0.7493} \cdot (CEP80)}{(20 \times 330) \cdot (MTHRU \times 1000)}$	$\frac{8.828E6 \cdot (MTHRU)^{0.7283} \cdot (CEP80)}{(20 \times 330) \cdot (MTHRU \times 1000)}$
	$=1.4930 \cdot (MTHRU)^{-0.2507} \cdot (CEP80)$	$=1.3376 \cdot (MTHRU)^{-0.2717} \cdot (CEP80)$
MINE OP	$22.12 + 3.2E-4 \cdot (DEPTH) \cdot (WPI80)$	$12.56 + 2.133E-4 \cdot (DEPTH) \cdot (WPI80)$
MILL OP	$\frac{\ln\left[\frac{(MTHRU)}{(12.9331)}\right] \cdot (WPI80)}{-0.25287}$	$\frac{\ln\left[\frac{(MTHRU)}{(13.0799)}\right] \cdot (WPI80)}{-0.27868}$
ROYLTY	2.0	2.5
HAUL	0	0
AVT	0	0
SEV	0	0

1) Depth in ft
2) MTHRU in 1000's tons/day
3) MSI80 = Marshall & Swift Index relative to 1/1/80
4) CEP80 = Chemical Engineering Index relative to 1/1/80
5) WPI80 = Wholesale Price Index relative to 1/1/80

VII. Haulage Cost Model (based on 10/78 Seminar Paper)

Open Pit, Underground Mines

Haul = Haul77 $\cdot$ WPI77

Region	Haul77
09,12	$ .90/ton
06,07	1.20
01,03,05,13,14,15,16	1.80
04,08,10,11,17,18,19,20	2.00
02	5.00

Solution Mines

Haul = $0.0/ton

1) Haul77 = Regional haulage costs, $1977
2) WPI77 = Wholesale Price Index relative to 1/1/77

VIII. Royalty Cost Model

A. Federal and Indian Lands:
(Land types 10,11,12,13,14,15,16,17,20,25,30)

Cost	Cost Category		
ROYLTY	$30	[A + BX] $\cdot$ X	X = UPRICE x GRADE 30 x 2000
	$50	[A + BX] $\cdot$ X	X = UPRICE x GRADE 50 x 2000
	$100	[A + BX] $\cdot$ X	X = UPRICE x GRADE 100 x 2000

A = 0.06   B = 0.0005

1) UPRICE = market value of U_3O_8 in concentrate, $/lb
2) GRADE XX = assumed average grade of ore in $XX/lb category
3) A, B = constants in Federal formula

B. State Lands:
(Land type 40)

For state codes: 42,25,49,40,43,05,30
all others, ROYLTY = $0.0/T

Calculate Price Adjusted Value of the Ore, OREVAL

OREVAL	Cost Category	
OREVAL 30	\$30	$\frac{\$2.5 \times 2000 \times UPRICE \times GRADE\ 30}{\$8}$
OREVAL 50	\$50	$\frac{\$1.5 \times 2000 \times UPRICE \times GRADE\ 50}{\$8}$
OREVAL 100	\$100	$\frac{\$0.50 \times 2000 \times UPRICE \times GRADE\ 100}{\$8}$

STATE 42 (TX)

Cost	Cost Category	
ROYLTY	\$30	0.14 x OREVAL 30
	\$50	0.13 x OREVAL 50
	\$100	0.13 x OREVAL 100

STATE 25 (MT) AND 49 (WYO)

Cost	Cost Category	
ROYLTY	\$30	0.05 x OREVAL 30
	\$50	0.05 x OREVAL 50
	\$100	0.05 x OREVAL 100

STATE 40 (SD)

Open Pit Mines

Cost	Cost Category		
ROYLTY	\$30	$[A + BX] \cdot X$	X = OREVAL 30
	\$50	$[A + BX] \cdot X$	X = OREVAL 50
	\$100	$[A + BX] \cdot X$	X = OREVAL 100

A = 0.0425 B = 0.0025

Underground Mines (SD)

Cost	Cost Category	
ROYLTY	\$30	$3.52\ (\text{OREVAL } 30)^{1.4342}$
	\$50	$3.52\ (\text{OREVAL } 50)^{1.4342}$
	\$100	$3.52\ (\text{OREVAL } 100)^{1.4342}$

STATE 43 (UTAH)

Cost	Cost Category		
ROYLTY	\$30	$[A + BX] \cdot X$	X = OREVAL 30
	\$50	$[A + BX] \cdot X$	X = OREVAL 50
	\$100	$[A + BX] \cdot X$	X = OREVAL 100

A = 0.028 B = 0.00306

STATE 05 (COLO)

Cost	Cost Category	
ROYLTY	\$30	(0.04) • (2000) x GRADE 30 x (UPRICE + ERDA)/2
	\$50	(0.04) • (2000) x GRADE 50 x (UPRICE + ERDA)/2
	\$100	(0.02) • (2000) x GRADE 100 x (UPRICE = ERDA)/2

ERDA = ERDA PRICE

STATE 30 (NM)

Cost	Cost Category	
ROYLTY	\$30	$0.125 \times \left[\text{UPRICE} - \frac{(\text{HAUL} + \text{MILLOP} + \text{MILCAP})}{(\text{GRADE30} \times \text{MR30} \times 2000)}\right] \times \text{GRADE } 30 \times 2000$
	\$50	$0.125 \times \left[\text{UPRICE} - \frac{(\text{HAUL} + \text{MILLOP} + \text{MILCAP})}{(\text{GRADE50} \times \text{MR50} \times 2000)}\right] \times \text{GRADE } 50 \times 2000$
	\$100	$0.125 \times \left[\text{UPRICE} - \frac{(\text{HAUL} + \text{MILLOP} + \text{MILCAP})}{(\text{GRADE100} \times \text{MR100} \times 2000}\right] \times \text{GRADE } 100 \times 2000$

C. Private and Other Lands
(Land types 50,90)

Cost	Cost Category	
ROYLTY	$30	$0.1 \times \left[UPRICE - \frac{(HAUL + MILLOP + MILCAP)}{(GRADE30 \times MR30 \times 2000)}\right] \times GRADE\ 30 \times 2000$
	$50	$0.1 \times \left[UPRICE - \frac{(HAUL + MILLOP + MILCAP)}{(GRADE50 \times MR50 \times 2000)}\right] \times GRADE\ 50 \times 2000$
	$100	$0.1 \times \left[UPRICE - \frac{(HAUL + MILLOP + MILCAP)}{(GRADE100 \times MR100 \times 2000)}\right] \times GRADE\ 100 \times 2000$

IX. Ad Valorem Tax Models

STATE 30 (NM)

Cost	Cost Category	
AVT	$30	0.06 x 0.25 x OREVAL 30 x MILREC 30
	$50	0.06 x 0.25 x OREVAL 50 x MILREC 50
	$100	0.06 x 0.25 x OREVAL 100 x MILREC 100

1) OREVALXX = adjusted value of ore, as in royalty model

2) MILRECXX = assumed mill recovery from ore in $XX/lb cost category

For all other states:

AVT = $0.0/ton

X. Severance Tax Models

STATE 30 (NM)

Cost	Cost Category	
SEV	$30	$3.27
	$50	$6.86
	$100	$3.43

STATE 49 (WYO)

Cost	Cost Category	
SEV	$30	0.055 x OREVAL 30
	$50	0.055 x OREVAL 50
	$100	0.055 x OREVAL 100

For all other states:

SEV = $ 0.0/ton

ATTACHMENT B

Obtaining the Distribution of Economic Potential from Uranium Endowment Estimates

The procedure described below has been developed to generate the distribution of economic potential from uranium endowment estimates and certain cost information. The model is a generalization of a manual procedure which has been used to produce a point-estimate of the economic potential in a specified cost category. The model assumes that economic factors determine which grades of material can be recovered at a desired cost per pound. The key mechanism for relating the information about economical grades to the endowment distribution is the curve describing cutoff grade versus average-grade-beyond-the-cutoff grade, developed from the mineral inventory analysis of the control areas.

Consider a favorable area for which the following have been estimated.

A, the area of the region (in square miles);
F, the fraction of the area underlain by mineralization;
T, the tonnage (in tons of uranium-bearing rock per square mile); and
G_e, the average grade of the mineralized rock above a cutoff grade of 0.01 percent U_3O_8.

Then U_e, the uranium endowment (in tons U_3O_8) is

$$U_e = A \cdot F \cdot T \cdot G_e/100. \quad (1)$$

Define the tons of mineralized rock as

$$TMR = A \cdot F \cdot T. \quad (2)$$

In order to estimate the economic potential, i.e., the in-place tons of U_3O_8 recoverable at a specified cost per pound, the following additional information is employed:

C_i, the cost index (in dollars per pound, e.g., $30/lb);
MR_i, the mill recovery rate (a fraction between 0 and 1);
OC_i, the total operating cost (in dollars per ton); and
CC_i, the total capital cost (in dollars per ton).

The model requires a function relating cutoff grade and average-grade-beyond-cutoff grade. One assumption*, based on DOE mineral inventory analysis, is that the average grade is a linear function of the cutoff grade, i.e.,

$$g = m(c-0.01) + G_e \quad (3)$$

*Although linear regressions seem to produce excellent approximations to these curves, the deviations of the real data from the "best" line appear to be systematic rather than random, especially at low cutoff grades. The model currently uses a parabolic segment between cutoff grades of 0.01 to 0.04 percent, and a linear segment above a 0.04 percent cutoff grade to reflect this phenomenon.

where g is the average grade of ore exceeding grade c. Note that for c = 0.01, $g = G_e$, the average grade of the endowment. The slope of the line is m.

Using this linear relationship, and the long established formulas for determining the economic cutoff and average grades;

$$c = \frac{OC_i}{C_i \cdot MR_i \cdot 20} \qquad \text{and} \qquad g = \frac{OC_i + CC_i}{C_i \cdot MR_i \cdot 20}$$

the cutoff grade is defined as the larger of:

$$c = OC_i/(20 \times MR_i \times C_i)$$

or

$$c = 0.01 + [(OC_i + CC_i)/(20 \times MR_i \times C_i) - G_e]/m \qquad (4)$$

This is the minimum ore grade which could be mined for C_i dollars per pound, and still assure that both the operating and capital costs are recovered.

A differentinal equation can be derived from equation (3) whose solution, F(c), is the probability that the grade of ore exceeds c. In other words, F(c) is the fraction of the mineral inventory having grades of at least c percent. Then the in-place economic potential, i.e., the tons of U_3O_8 recoverable at C_i dollars per pound, is

$$UC_i = TMR \cdot F(c) \cdot g/100.$$

Recall that TMR is the tons of mineralized rock (having grade at least 0.01 percent); F(c) is the fraction of that rock having grades exceeding the required cutoff grade, c; and g is the average grade of this fraction of the total mineral inventory. Note that TMR is independent of economic considerations, while both F(c) and g depend on G_e, c, and, hence, C_i, OC_i, CC_i; and MR_i.

In the estimation of endowment and economic potential, the quantities F, T, and G_e are random variables whose distributions have been (subjectively) described by the estimator. In this case, F(c) and g are also random variables whose distributions may be completed from the information available about G_e and c.